# A semi-empirical theory for forced-flow turbulent film boiling of subcooled liquid along a horizontal plate

#### BU-XUAN WANG and DE-HUI SHI

Thermal Engineering Department, Tsinghua University, Beijing 1000084, China

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Abstract — Based on a previous study [1], a physical model is proposed in this paper to analyze the film boiling heat transfer for turbulent flow of subcooled liquid along a horizontal plate. The corresponding mathematical descriptions are given in detail and an analytical expression has been obtained as equation (28), which is simple in form. The empirical values of coefficient k and exponent m are determined from experiments with subcooled water. As a result, equation (30) will be adaptable for practical use in engineering analysis of high-temperature quenching process for rolled metals.

#### 1. INTRODUCTION

IN A PREVIOUS paper [1], analyses were made for the film boiling on a horizontal plate surface in the laminar boundary layer of a liquid greatly deviating from its critical state. The effects of both the pressure gradient caused by an increase in the vapor-film thickness along the flow direction and the subcooling of liqid on heat transfer had been discussed. However, the laminar flow will change over to turbulent at a certain critical distance from the leading edge along flow passage. which is shortened as the free-stream velocity increases. As shown by Suryanarayana [2], the laminar film boiling with a steady vapor-liquid interface is limited to a comparatively small distance from the leading edge of the plate; and in most cases, there will exist fluctuations at the two-phase interface, which direct the characteristic of turbulent film boiling from the start.

In the 1980s the development in efficient coolingcontrol technology of high-temperature quenching process, which generally corresponds to the use of forced-turbulent film boiling, has been regarded as one of the more important technical duties in the metallurgy industry. Although several industrial measurement data on the quenching of rolled metal sheets have been reported in literature, the correlations of heat transfer data confined to local conditions, as recommended by different researchers, differ so greatly that it is difficult to adapt them in practical use. We still know very little about the characteristic of such twophase flow heat transfer, especially for the subcooled liquid flowing at higher velocity [3]. As a result, it is usual to calculate the heat transfer rate for the turbulent film boiling of subcooled liquid flowing at higher velocity with the well-known empirical equations correlating the heat transfer data for the single-phase turbulent flow [4]. An analysis of subcooled film boiling inside a horizontal duct was recently reported by Viannay and Karian [5], but the assumption that all the heat transferred from the hightemperature plate to the opposite cold plate of the duct may be not true, and the subcooled liquid flow will actually carry away most of the heat taken from the high-temperature plate of the duct.

On the basis of analyzing the previous paper [1], a physical model will be proposed in this paper for the turbulent film boiling on a horizontal plate, and the corresponding mathematical descriptions are established to analyze theoretically the heat transfer characteristics for turbulent flow of subcooled liquid along a horizontal plate.

### 2. CONCEPTION OF THE PHYSICAL MODEL

As the laminar film boiling of liquid deviates greatly from its critical state, i.e.  $(\rho\mu)_2/(\rho\mu)_1\gg 1$ , the velocity profile approaches uniformity within the liquid region as obtained in the previous paper [1], and the velocity gradient occurs mostly in the vapor film. This is the same result as that reported by Ito and Nishikawa [6] who solved numerically two-phase boundary-layer equations.

With the thermophysical properties of liquid being considered as constants, the energy equation in the liquid region for laminar flow will be

$$u_2 \frac{\partial t_2}{\partial x} + v_2 \frac{\partial t_2}{\partial y} = a_2 \frac{\partial^2 t_2}{\partial y^2}.$$
 (1)

Assuming uniform velocity distribution in the liquid region, i.e. taking  $u_2 = u_{\infty}$  and  $v_2 = 0$  as the first approximation, by changing the ordinate referring to the liquid-vapor interface so that y' = 0 at  $y = \delta$ , equation (1) is then reduced to

$$u_{\infty} \frac{\partial t_2}{\partial x} = a_2 \frac{\partial^2 t_2}{\partial y'^2}.$$
 (2)

Since  $u_{\infty}$  is independent of x, equation (2) can be

#### **NOMENCLATURE**

a thermal diffusivity

c<sub>p</sub> specific heat at constant pressure

 $h_{\rm fg}$  latent heat of evaporation

Nu Nusselt number

Pr Prandtl number

q local heat transfer rate per unit area

Re Reynolds number

t temperature

u, v velocity component in x- and y-directions, respectively.

Greek symbols

α local heat transfer coefficient

 $\delta$  thickness of vapor film

λ thermal conductivity

 $\rho$  density

 $\mu$  absolute viscosity

v kinematic viscosity.

Subscripts

w at surface

∞ at infinite distance from plate surface

s saturated condition

1 vapor

2 liquid.

rewritten as follows:

$$\frac{\partial t_2}{\partial (x/u_{\infty})} = a_2 \frac{\partial^2 t_2}{\partial y'^2}.$$
 (3)

The boundary conditions will be:

for 
$$x = 0$$
, (or  $\tau = x/u_{\infty}$ ),  $t_2 = t_{\infty}$   
for  $y' = 0$ ,  $t_2 = t_s$   
 $y' = \infty$ ,  $t_2 = t_{\infty}$ .

 $x/u_{\infty}$  has the dimension of time and may be regarded as the time required for the liquid, flowing at velocity  $u_{\infty}$ , to go from x=0 to x=x. So, it is analogous to the unsteady heat conduction for a semi-infinite body with uniform initial temperature  $t_{\infty}$  while its surface temperature changes suddenly from  $t_{\infty}$  to  $t_{\rm s}$ , and the corresponding solution is well-known as [7]:

$$\frac{t_s - t_2}{t_s - t_\infty} = \operatorname{erf}\left(\frac{y'}{2\sqrt{a_2 x/u_\infty}}\right). \tag{5}$$

From Fourier's law, the heat flux transferred from the vapor-liquid interface to the liquid would be

$$q_1 = -\lambda_2 \frac{\partial t_2}{\partial y'}\Big|_{y'=0} = \frac{1}{\sqrt{\pi}} \lambda_2 (t_s - t_\infty) \sqrt{\frac{u_\infty}{a_2 x}}.$$
 (6)

For the case of liquid subcooled greatly, the increase of vapor-film thickness is suppressed strongly [8], and all of the heat flux from the wall,  $q_{\rm w}$ , will be transferred to the liquid region, i.e.  $q_1=q_{\rm w}$ . Hence, if let

$$\widetilde{Nu}_{x} = \frac{q_{w}}{t_{s} - t_{\infty}} \frac{x}{\lambda_{2}},$$

a revised local Nusselt number based on the subcooled temperature  $(t_s - t_\infty)$ , it is easily obtained from equation (6) so that

$$\widetilde{Nu}_x = \frac{1}{\sqrt{\pi}} \sqrt{\frac{u_\infty x}{v_2}} = \frac{1}{\sqrt{\pi}} Re_2^{1/2},$$
 (7)

which is just the same as the heat transfer relation derived more rigorously in the previous paper [1].

We conclude therefore that, for the laminar film boiling of a subcooled liquid deviating greatly from its critical state, the velocity profile within the liquid region can be regarded as being uniform because of great difference between vapor and liquid density. For the case of subcooled film boiling in forced-turbulent flow, the velocity distribution within the liquid region will be more even owing to strong turbulent mixing combined with evaporation of liquid and condensation of vapor at the liquid—vapor interface. Hence, as shown in Fig. 1, the basic assumptions adopted are as follows:

- (1) It is reasonable to divide the whole flow pattern into three regions, i.e. the vapor layer near the wall surface, the liquid flow region and an intermediate vapor-liquid mixing region.
- (2) The 'vapor-liquid mixing region' is the region composed of vapor bubble flow and fluctuating liquid-vapor interface, the thickness of this region depends on free-stream velocity,  $u_{\infty}$ , and degree of liquid temperature subcooling,  $(t_s t_{\infty})$ . The vapor-liquid mixing process is expected to be so strong that not only will the time-mean velocity distribution within this mixing region be uniform, but also the time-mean temperature within this region can be considered as being constant and equal to saturated temperature,  $t_s$ .
- (3) The time-mean velocity distribution within the liquid flow region is uniform and kept constant as free-

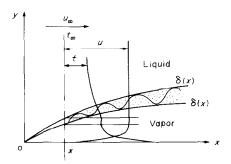


Fig. 1. Analytical model.

stream velocity,  $u_{\infty}$ , so that this region can be regarded as the turbulent main stream region beyond the turbulent boundary layer.

(4) The turbulent boundary layer would then coincide with vapor-film layer, because the time-mean velocity distribution in the vapor-liquid mixing region has been assumed uniform and will be equal to  $u_{\infty}$  also. Therefore, the flow characteristic in the vapor-film region will be the same as that of single-phase turbulent flow along a horizontal plate surface, and both the universal velocity profile proposed by Kármán and the analogy between momentum and heat transfer [7] should hold true. So, as observed by Bradfield [9], the wall friction for turbulent-flow film boiling is obviously lower than that for a single-phase turbulent liquid flow, since viscosity of vapor is much smaller than that of liquid.

#### 3. MATHEMATICAL FORMULATION

According to the physical model suggested, the energy equation in liquid flow region should be:

$$u_{\infty} \frac{\partial t_2}{\partial x} = \frac{\partial}{\partial y} \left[ (a_2 + \varepsilon_{t,2}) \frac{\partial t_2}{\partial y} \right]$$
 (8)

where  $t_2$  is the time-mean temperature of liquid,  $\varepsilon_{t,2} = \lambda_{t,2}/(\rho_2 c_{\rm p2})$  is the turbulent thermal diffusivity and  $\lambda_{t,2}$  is the turbulent thermal conductivity of liquid.

Noting,  $\varepsilon_{t,2} \gg a_2$  and  $\lambda_{t,2} \gg \lambda_2$  for the violent turbulent liquid flow. Taking conventionally  $Pr_t = \varepsilon_{\rm M}/\varepsilon_t \approx 1$ , where  $\varepsilon_{\rm M}$  is the 'turbulent viscosity for momentum exchange', equation (8) can be simplified to

$$u_{\infty} \frac{\partial t_2}{\partial x} = \frac{\partial}{\partial y} \left[ \varepsilon_{\mathbf{M}, 2} \frac{\partial t_2}{\partial y} \right]. \tag{9}$$

From Kármán's Theory, it can be written that

$$\varepsilon_{\mathbf{M}}/\mathbf{v} = f(\mathbf{y}^+) \tag{10}$$

where  $y^+ = \sqrt{s_{\mathbf{w},\mathbf{x}}/\rho} \cdot y/v$  is a modified Reynolds number with shear-stress velocity  $v_{\mathbf{f}} = \sqrt{s_{\mathbf{w},\mathbf{x}}/\rho}$ . For the forced-turbulent film boiling, the increase in the vapor-film thickness along the flow direction will intensify the phase-transition process within the liquid-vapor mixing region, meanwhile, fluctuations at the vapor-liquid interface as well as vortex movement of liquid will spread rapidly into the liquid region, so that the turbulent viscosity of liquid becomes very large and approximately uniform at any given x-section, i.e. the turbulent viscosity may be independent of y at  $y = \delta'_x$ . However,  $\delta'_x$  should depend on x, and  $s_{\mathbf{w},\mathbf{x}}/\rho$  may be a function of  $Re_x$ . Hence, for the liquid region, equation (10) can be expressed as:

$$\varepsilon_{\rm M, 2}/v_2 = k Re_x^{\rm m} = k(u_{\infty}x/v_2)^{\rm m}.$$
 (11)

The dimensionless coefficient k in equation (11) may be a weak function of the physical properties of liquid and can be taken as an empirical constant to be determined by experiments. The exponent m depends on fluctuation of the liquid-vapor interface and may be a

weak function of the time-mean vapor-layer thickness, i.e.  $m = f(\delta_x/x)$ .

It is found from experimental observations that, because of the strong fluctuations at the vapor-liquid interface as well as the suppression of subcooled liquid flow, the thickness of vapor-film layer in the x-direction increases so slowly that exponent m could approach a constant. As a special case, if m is taken as 1, equation (11) will be reduced to  $\varepsilon_{\rm M} \sim (u_{\infty}x)$ , which corresponds to the single-phase free-shear-layer flow as analyzed by Eckert [10].

Substituting equation (11) into (9), and changing the coordinate referring to the vapor-liquid interface, i.e. taking y' = 0 at  $y = \delta'_x$ , the following equation is obtained:

$$u_{\infty} \frac{\partial t_2}{\partial x} = k v_2 \left( \frac{u_{\infty} x}{v_2} \right)^m \frac{\partial^2 t_2}{\partial v^2}, \tag{12}$$

with boundary conditions

$$x = 0, t_2 = t_{\infty}$$
  
 $y' = 0, t_2 = t_{s}$   
 $y' = \infty, t_2 = t_{\infty}$ . (13)

So long as the values of k and m are known, the temperature profile and the temperature gradient at y' = 0 will be obtained from equations (12) and (13) in combination, and the heat flux can be then calculated from Fourier's Law.

For the vapor-film region, introducing Kármán's universal velocity distribution and the analogy between momentum and heat transfer, together with boundary conditions

$$y = 0,$$
  $t_1 = t_{w,x}$   
 $y = \delta_x,$   $t_1 = t_s$  (14)

the following familiar correlation can be obtained [7]:

$$\frac{t_{w,x} - t_{s}}{q_{w,x}} = \frac{u_{\delta_{x}} / \sqrt{s_{w,x}/\rho_{1}} + 5(Pr - 1) \ln \left[ (5Pr + 1)/6 \right]}{\rho_{1} c_{p_{1}} \sqrt{s_{w,x}/\rho_{1}}}$$
(15)

where  $s_{w,x}$  is the shear stress on wall surface. Using Blasius' formula for the friction coefficient  $c_{f,x}$  of single-phase flow, we get

$$s_{w,x} = \frac{1}{2}\rho_1 u_{\infty}^2 c_{f,x} = 0.023 \left(\frac{u_{\infty} \delta_x}{\rho_1}\right)^{-1/4} \rho_1 u_{\infty}^2.$$
 (16)

For most of liquids, including water, deviating greatly from their critical states, Prandtl numbers for their vapors are close to 1, equation (15) can be thus reduced to

$$\frac{t_{w,x} - t_s}{q_{w,x}} = \frac{u_{\delta_x} / \sqrt{s_{w,x} / \rho_1}}{\rho_1 c_{p_1} \sqrt{s_{w,x} / \rho_1}} = \frac{u_{\infty}}{c_{p_1} s_{w,x}}.$$
 (17)

Meanwhile, the turbulent velocity profile within

vapor film can be approximately expressed as

$$\frac{u_1}{u_\infty} = \left(\frac{y}{\delta_x}\right)^{1/7}.\tag{18}$$

As shown in Fig. 1, there exists a balance relation of heat flux according to conservation of energy:

$$q_{w,x} = q_{1,x} + q_{g,x} \tag{19}$$

where  $q_{g,x}$  is the heat flux used to evaporate the liquid, and can be given by

$$q_{g,x} = \rho_1 h_{fg} \frac{d}{dx} \int_0^{\delta_x} u_1 dy.$$
 (20)

Substituting equation (18) into equation (20), we get

$$q_{\mathbf{g},x} = \frac{7}{8}\rho_1 h_{\mathbf{f}\mathbf{g}} u_{\infty} \frac{\mathrm{d}\delta_x}{\mathrm{d}x}.$$
 (21)

Substituting  $q_{w,x}$  from equation (17),  $q_{g,x}$  from equation (21) and  $q_{l,x}$  calculated from equation (12) with boundary condition (13) into equation (19), we obtain an equation with known quantities and x. Solving as a function of x, the heat transfer flux on wall surface can be finally determined.

#### 4. FILM BOILING HEAT TRANSFER FOR TURBULENT FLOW OF SUBCOOLED LIQUID ALONG A HORIZONTAL PLATE

Introducing a dimensionless subcooled temperature,  $\theta = (t_s - t_2)/(t_s - t_\infty)$ , equation (12) is rewritten as

$$u_{\infty} \frac{\partial \vartheta}{\partial x} = k v_2 \left( \frac{u_{\infty} x}{v_2} \right)^m \frac{\partial^2 \vartheta}{\partial y'^2}. \tag{22}$$

Let  $\eta = (u_{\infty}x/v_2)^{m-1}x^2$ , with dimension of  $m^2$ . Substituting into equation (22), we get

$$\frac{\partial \theta}{\partial \eta} = \frac{k}{m+1} \frac{\partial^2 \theta}{\partial y'^2}.$$
 (23)

From equation (13), the corresponding boundary conditions are:

$$\eta = 0, \quad \vartheta = 1 
y' = 0, \quad \vartheta = 0 
y' = \infty, \quad \vartheta = 1.$$
(24)

The solution of equations (23) and (24) is given as:

$$\vartheta = \operatorname{erf} \frac{y'}{2\sqrt{\frac{k}{m+1}}\eta} \quad , \tag{25}$$

or

$$\theta = \operatorname{erf}\left(\frac{y'}{2\sqrt{\frac{k}{m+1}}(u_{\infty}x/v_{2})^{(m-1)/2}x}\right). \tag{26}$$

Hence, from Fourier's law,

$$q_{1,x} = \rho_2 c_{p_2} \varepsilon_{M,2} (t_s - t_{\infty}) \frac{\partial \vartheta}{\partial \nu'} \Big|_{\nu'=0}$$

or

$$q_{1,x} = \sqrt{\frac{k(m+1)}{\pi}} \left( t_{s} - t_{\infty} \right) \pi_{2} c_{p_{2}} v_{2} \frac{1}{x} \left( \frac{u_{\infty} x}{v_{2}} \right)^{(m+1)/2}.$$
(27)

Solving numerically equations (16), (17), (19), (21) and (27), it is found that, as being pointed out previously by Kalinin [4] for the turbulent flow of subcooled liquid, the film boiling heat transfer process are mainly governed by physical properties of liquid rather than those of vapor, and seldom influenced by the wall surface temperature. It is therefore confirmed that heat flux on the wall surface is mostly transferred to the liquid flow, i.e.  $q_{1,x} \approx q_{w,x}$ . Noting  $Pr_2 = \rho_2 c_{p_2} v_2/\lambda_2$ , the local revised Nusselt number

$$\widetilde{Nu}_{x} = \frac{q_{w,x}}{t_{s} - t_{\infty}} \frac{x}{\lambda_{2}}$$

can be obtained directly from equation (27) as

$$\widetilde{Nu}_x = k' Re_x^{(m+1)/2} Pr_2.$$
 (28)

where

$$k' = \sqrt{\frac{k(m+1)}{\pi}} \,. \tag{29}$$

It is obvious that equation (28) is only available to the turbulent film boiling for subcooled liquid flowing along a horizontal plate. However, since the film boiling for turbulent flow of subcooled liquid is seldom influenced by the wall temperature, there is no doubt that equation (28) can be used to calculate the heat transfer rate in quenching process for rolled metal at high temperature.

It is clear from equation (27) that  $q_1$  decreases (causing an increase in  $q_8$ ) with the decrease in the degree of subcooling for the liquid flow  $t_s - t_\infty$ . When the temperature of flowing liquid gradually approaches its saturation temperature,  $q_8$  cannot be ignored compared with  $q_1$ , and equation (28) does not then hold true. For the case of saturated flow film boiling,  $q_1$  should be zero as the limit, and the heat transfer would be mainly governed by physical properties of vapor, rather than liquid, and the flow characteristic within vapor film. Such a case will be discussed later.

#### 5. EXPERIMENTAL CHECK

As an analytical solution, equation (28) needs to be verified by experiments. This was done to ensure that the measured data can converge to a straight line in a log  $(\widetilde{Nu}_x/Pr_2)$  log  $Re_x$  plot. If it does, the rate of inclination for the straight line will be (m+1)/2 and the ordinate for the straight line extending to log  $Re_x = 0$ 

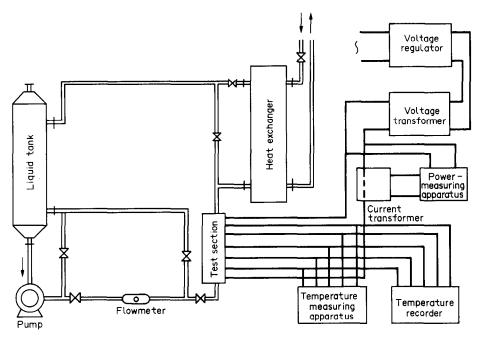


Fig. 2. The schematic diagram of experimental unit.

will be  $\log k'$ , then the values of k and m can be easily determined.

Experimental data on film boiling heat transfer for turbulent flow of subcooled liquid, especially water, along a horizontal plate has been reported rarely. Therefore, an experimental investigation was made for subcooled deionized water, flowing at atmospheric pressure, with velocity in the range 1-4.5 m s<sup>-1</sup> and with temperature subcooled range 22-72°C. Figure 2 shows the layout of the experimental installation. The test section is made of a short horizontal flat duct, with a test copper plate 200-mm long, 80-mm wide and 8-mm thick as the bottom plate. When the test plate is heated, by the electric heater mounted below, to reach 650-700°C the cooling water flows suddenly through the test section. The surface temperatures of the test plate, in response of the cooling, are measured by thermocouples at 50 and 150 mm from the front edge, respectively, and are recorded automatically, so that the heat flux  $q_{w,x}$  can be calculated. The measuring techniques and the reliability of the data taken from experiments has been discussed in detail [11]. It is found that the wall surface heat flux is really independent of the surface temperature as was predicted analytically.

Typical experimental data are plotted in Fig. 3, and the data points converge to a straight line as indicated by equation (28) with k = 0.0055 and m = 0.68. The maximum deviation for all the data points, taking from x = 50 and 150 mm, did not exceed  $\pm 25\%$ . Hence, equation (28) can be rewritten as

$$\widetilde{Nu}_x = 0.054 \ Re_x^{0.84} \ Pr_2. \tag{30}$$

Since equation (28) is derived from the physical model suggested, the higher the flow velocity and the greater the degree of temperature subcooling for the liquid flow, the more the basic assumptions tend towards reality. So, equation (30) can be extended to predict the heat transfer rate in the high-temperature quenching process of rolled metal sheets, especially for water with greater degree of temperature subcooled and flowing at higher velocity. However, k = 0.0055 and m = 0.68 are obtained from experiments with subcooled water only, these values cannot be regarded as universal constants according to the theory. Therefore, the suitability of equation (30) to film boiling for turbulent flow of subcooled liquids other than water should be checked by further experiments.

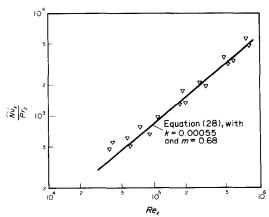


Fig. 3. Plot of experimental data for turbulent-flow film boiling of subcooled water.

#### 6. CONCLUSIONS

The present work may be summarized as follows:

- (1) An essentially physical model, as shown in Fig. 1, is presented to describe the process of turbulent film boiling for subcooled liquid, and the analytical solution, equation (28), thus derived has been verified by experimental data for water. The model, being simple in form and convenient in use, can be regarded as an effective tool for analyzing the film-boiling heat transfer for turbulent flow of subcooled liquid.
- (2) For the film boiling in violent turbulent flow, the turbulent diffusivity of momentum exchange,  $\varepsilon_{\rm M.2}$  within the liquid flow region is to be expressed in a more general dimensionless form of equation (11). As a special example, if the value of m is taken as 1, equation (11) will be reduced to the simple form of  $\varepsilon_m \sim (u_\infty x)$ , which corresponds to the case of single-phase free-shear-layer flow. Since the fluctuations of vapor-liquid interface would affect the liquid flow, it is more complex than the single-phase free-shear-layer flow, and the actual value of m for subcooled water film boiling is not 1, but approaches 0.68 as determined experimentally.
- (3) Equation (28) is practically suitable to the turbulent film boiling on a horizontal plate for the subcooled liquid deviating greatly from its critical state. For subcooled water, k = 0.0055 and m = 0.68 are obtained experimentally, thus equation (30) can be used to predict heat transfer rate for quenching of rolled metal sheets at high temperature by water flowing with much higher velocity.
- (4) Further experiments will still be necessary to check the suitability of equation (28) as well as to

determine the values of k and m for liquids other than water.

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## UNE THEORIE SEMI-EMPIRIQUE POUR L'EBULLITION EN FILM AVEC ECOULEMENT TURBULENT FORCE D'UN LIQUIDE SOUS-REFROIDI LE LONG D'UNE PLAQUE HORIZONTALE

Résumé—A partir d'une étude antérieure [1], un modèle physique est proposé pour analyser le transfert thermique de l'ébullition en film pour un écoulement turbulent de liquide sous-refroidi le long d'une plaque horizontale. Les descriptions mathématiques sont données en détail et une expression analytique telle que l'équation (28) est obtenue dans une forme simple. Les valeurs empiriques du coefficient k et de l'exposant m sont déterminés à partir d'expériences avec de l'eau sous-refroidie. Il en résulte que l'équation (30) est adaptable à l'ingéniérie pour la trempe des métaux en feuille à haute température.

#### EINE HALB-EMPIRISCHE THEORIE FÜR DAS TURBULENTE FILMSIEDEN BEI ERZWUNGENER STRÖMUNG VON UNTERKÜHLTER FLÜSSIGKEIT LÄNGS EINER HORIZONTALEN PLATTE

Zusammenfassung—In dieser Arbeit wird—ausgehend von einer früheren Studie [1]—ein physikalisches Modell vorgeschlagen, um den Wärmeübergang beim Filmsieden für eine turbulente Strömung einer unterkühlten Flüssigkeit längs einer horizontalen Platte zu analysieren. Die zugehörigen mathematischen Gleichungen wurden in allen Einzelheiten erläutert, und man erhält einen analytischen Ausdruck nach Gleichung (28) von sehr einfacher Form. Die empirischen Werte des Koeffizienten k und des Exponenten m werden aus Versuchen mit unterkühlten Wasser ermittelt. Die Ergebnisse können bei der ingenieurmäßigen Auslegung des Abschreck-Vorgangs von gewalztem Metall von hoher Temperatur von praktischem Nutzen sein.

## ПОЛУЭМПИРИЧЕСКАЯ ТЕОРИЯ ВЫНУЖДЕННОГО ТУРБУЛЕНТНОГО ПЛЕНОЧНОГО КИПЕНИЯ НЕДОГРЕТЫХ ЖИДКОСТЕЙ ВДОЛЬ ГОРИЗОНТАЛЬНОЙ ПЛАСТИНЫ

Аннотация—На основе ранее проведенного исследования [I] в данной работе предложена аналитическая модель для анализа теплообмена при пленочном кипении для турбулентного течения недогретой жидкости вдоль горизонтальной пластины. Приводится детальное математическое описание, также получено простое по форме аналитическое выражение, подобное уравнению (28). Эмпирические значения коэффициента k и показателя экспоненты m определяются из экспериментов с недогретой водой. В результате уравнение (30) пригодно для практического использования при анализе высокотемпературного процесса закалки прокатываемых металлов.